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LR Scalar Mixings and One-loop Neutrino Masses

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Abstract

Within the framework of the complete theory of supersymmetry without R-parity, where all possible R-parity violating terms are admitted, we perform a systematic analytical study of all sources of neutrino masses up to one-loop level. In the passing, we present the full result for squark and slepton masses. In particular, there are interesting LR squark and slepton mixings which involve both bilinear and trilinear R-parity violating parameters and hence have been missing in previous studies under which either one type of the couplings is assume to vanish. The LR mixings play an central role in neutrino mass generations.

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Introduction : The minimal supersymmetric standard model (MSSM) is no doubt the most popular candidate theory for physics beyond the Standard Model (SM). The alternative theory with a discrete symmetry called R-parity not imposed deserves no less attention. In particular, the latter theory admits neutrino masses, without the need for any extra matter field beyond the minimal spectrum. Experimental results from neutrino physics [1] is actually the only data we have asking definitely for physics beyond the SM. The data provides strong hint at the existence of Majorana type mass contributions. The latter means lepton number violation, which is suggestive for R-parity violation. Hence, it is easy to appreciate the interest in R-parity violating (RPV) contributions to neutrino masses. Recent works in the subject area [2–5] emphasize mainly on fitting the neutrino oscillation data under different scenarios while a comprehensive analysis of all the RPV contributions is still missing. This letter aims at providing such a picture. Like most of the other studies, we will focus on the sub-eV neutrino mass scale suggested by the Super-Kamiokande atmospheric neutrino data [6], though most of our results are actually valid at a much larger range of neutrino masses. As illustrated below, there is a tree-level but see-saw suppressed contribution and many direct 1-loop contributions. Our level of treatment in this letter stops there, *i.e.* we will, in general, not go into contributions that are further suppressed. The latter includes 1-loop contributions which involve a further see-saw type suppression. Some of the results given here are scattered among earlier works, particularly some recent works of the author and other collaborators. Others are new. The idea here is to perform a systematic analysis and present all these in a complete and self-contained story.

A similar comprehensive listing of neutrino mass contributions up to 1-loop level has been presented in Ref. [7]. However, the latter analysis is limited to a scenario where the "third generation couplings dominate". Among the trilinear RPV couplings, this amount to admitting only non-zero λ'_{i33}'s and λ_{i33}'s, though all nonzero bilinear RPV are indeed included. In our opinion, the maximal mixing result from Super-Kamiokande [6] brings that wisedom of "third generation domination" into question. Refs. [3] and [5], for example illustrate how no (family) hierarchy, or even an anti-hierarchy, among the RPV couplings may be preferred. The present analysis handles the complete theory of supersymmetry (SUSY) without R-parity, where all kind of RPV terms are admitted without bias. There is another major difference between the two studies. Ref. [7] is interested in performing some numerical calculation. While the latter is important in explicit fitting of experimental numbers, much of the physical origin of the neutrino mass contributions are hidden under elements of mixing matrices parametrizing the effective couplings of the neutrinos to squark or slepton mass eigenstates. We are interested here in illustrating the explicit origin of each contribution. Hence, we stay with weak state notation and give diagrammatic as well as analytical expressions of each individual contribution. Of particular interest here is a new type of contributions involving a RPV LR scalar (squark or slepton) mixings, which has been missing in Ref. [7] and the other studies on various RPV scenarios [1]. It is the hope of the author that results here would be useful for better understanding the role of each RPV parameter and identifying interesting regions of the extensive parameter space.

To study all the RPV contributions in a single consistent framework, one needs an effective formulation of the complete theory of SUSY without R-parity. The latter theory is generally better motivated than *ad hoc* versions of RPV theories. The large number of new parameters involved, however, makes the theory difficult to analyze. It has been illustrated [8] that an optimal parametrization, called the single-VEV parametrization, can be of great help in making the task manageable. The effectiveness of the SVP has been explored to perform an extensive study on resultant leptonic phenomenology [8], to identify new type of neutrino mass contributions [5], and to study a new contribution to neutron electric dipole moment at 1-loop level [9], as well as new sources of contribution to flavor changing neutral current processes such as $b \to s\gamma$ [10]. Works on neutrino masses and mixings under the formulation also include Refs. [3,4]. In fact, neutrino masses contribution is a central aspect of RPV effects and is likely to provide the most stringent bounds on the couplings, though many of the bound obtained depend on assumptions on interpretation of neutrino data and could be relaxed or removed by simple extension of the theory allowing extra sterile neutrino(s).

One-loop neutrino mass generations in SUSY without R-parity typically involves LR mixings of squarks or slepton. We present, here in this letter, also the full results for squark and slepton masses. We consider the results to be interesting in their own right.

Formulation and Notations : We summarize our formulation and notations below. The most general renormalizable superpotential for the supersymmetric SM (without R-parity) can be written then as

$$W = \varepsilon_{ab}\left[\mu_\alpha \hat{H}_u^a \hat{L}_\alpha^b + h_{ik}^u \hat{Q}_i^a \hat{H}_u^b \hat{U}_k^C + \lambda'_{\alpha jk} \hat{L}_\alpha^a \hat{Q}_j^b \hat{D}_k^C + \frac{1}{2}\,\lambda_{\alpha\beta k} \hat{L}_\alpha^a \hat{L}_\beta^b \hat{E}_k^C\right] + \frac{1}{2}\,\lambda''_{ijk} \hat{U}_i^C \hat{D}_j^C \hat{D}_k^C \ , \quad (1)$$

where (a,b) are $SU(2)$ indices, (i,j,k) are the usual family (flavor) indices, and (α,β) are extended flavor index going from 0 to 3. At the limit where $\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$ and μ_i all vanish, one recovers the expression for the R-parity preserving case, with $\hat{L}_0$ identified as $\hat{H}_d$. Without R-parity imposed, the latter is not *a priori* distinguishable from the $\hat{L}_i$'s. Note that λ is antisymmetric in the first two indices, as required by the $SU(2)$ product rules, as shown explicitly here with $\varepsilon_{12} = -\varepsilon_{21} = 1$. Similarly, λ'' is antisymmetric in the last two indices from $SU(3)_C$.

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[1] Some contributions of the type were first identifed, within the slepton sector is, in a recent paper by K. Cheung and the present author [5], motivated in a different context.

R-parity is exactly an *ad hoc* symmetry put in to make $\hat{L}_0$, stand out from the other $\hat{L}_i$'s as the candidate for $\hat{H}_d$. It is defined in terms of baryon number, lepton number, and spin as, explicitly, $\mathcal{R} = (-1)^{3B+L+2S}$. The consequence is that the accidental symmetries of baryon number and lepton number in the SM are preserved, at the expense of making particles and superparticles having a categorically different quantum number, R-parity. The latter is actually not the most effective discrete symmetry to control superparticle mediated proton decay [11], but is most restrictive for in terms of what is admitted in the Lagrangian, or the superpotential alone.

A naive look at the scenario suggests that the large number of new couplings makes the task formidable. However, it becomes quite manageable with an optimal choice of flavor bases, the SVP [8]. In fact, doing phenomenological studies without specifying a choice of flavor bases is ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings instead of the 10 real physical parameter. For SUSY without R-parity, the choice of an optimal parametrization mainly concerns the 4 $\hat{L}_\alpha$ flavors. Under the SVP [2] , flavor bases are chosen such that : 1/ among the $\hat{L}_\alpha$'s, only $\hat{L}_0$, bears a VEV *i.e.* $\langle\hat{L}_i\rangle \equiv 0$; 2/ $h^e_{jk} (\equiv \lambda_{0jk} = -\lambda_{j0k}) = \frac{\sqrt{2}}{v_0}\mathrm{diag}\{m_1, m_2, m_3\}$; 3/ $h^d_{jk} (\equiv \lambda'_{0jk}) = \frac{\sqrt{2}}{v_0}\mathrm{diag}\{m_d, m_s, m_b\}$; 4/ $h^u_{ik} = \frac{v_u}{\sqrt{2}} V^\dagger_{CKM} \mathrm{diag}\{m_u, m_c, m_t\}$, where $v_0 \equiv \sqrt{2}\,\langle\hat{L}_0\rangle$ and $v_u \equiv \sqrt{2}\,\langle\hat{H}_u\rangle$. The big advantage of here is that the (tree-level) mass matrices for all the fermions do not involve any of the trilinear RPV couplings, though the approach makes no assumption on any RPV coupling including even those from soft SUSY breaking; and all the parameters used are uniquely defined, with the exception of some removable phases. In fact, the (color-singlet) charged fermion mass matrix is reduced to the simple form :

$$\mathcal{M}_c = \begin{pmatrix} M_2 & \frac{g_2 v_0}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v_u}{\sqrt{2}} & \mu_0 & \mu_1 & \mu_2 & \mu_3 \\ 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & m_3 \end{pmatrix} . \tag{2}$$

Each μ_i parameter here characterizes directly the RPV effect on the corresponding charged lepton ($\ell_i = e$, μ, and τ) respectively. For any set of other parameter inputs, the m_i's can then be determined, through a simple numerical procedure, to guarantee that the correct mass eigenvalues of m_e, m_μ, and m_τ are obtained — an issue first addressed and solved in Ref. [8]. The latter issue is especially important when μ_i's not substantially smaller than μ_0 are considered. Such an odd scenario is not definitely ruled out [8]. However, we would concentrate here on the more popular scenario with only sub-eV neutrino masses and hence

[2]Note that our notations here are a bit different form those in the reference.

small μ_i's.

Gauginos, Higgsinos, and Neutrinos : The tree-level mixing among the gauginos, higgsinos, and neutrinos gives rise to a 7×7 neutral fermion mass matrix $\mathcal{M}_{\mathcal{N}}$:

$$\mathcal{M}_{\mathcal{N}} = \left(\begin{array}{cccc|ccc} M_1 & 0 & g_1 v_u/2 & -g_1 v_0/2 & 0 & 0 & 0 \\ 0 & M_2 & -g_2 v_u/2 & g_2 v_0/2 & 0 & 0 & 0 \\ g_1 v_u/2 & -g_2 v_u/2 & 0 & -\mu_0 & -\mu_1 & -\mu_2 & -\mu_3 \\ -g_1 v_0/2 & g_2 v_0/2 & -\mu_0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -\mu_1 & 0 & (m^o_\nu)_{11} & (m^o_\nu)_{12} & (m^o_\nu)_{13} \\ 0 & 0 & -\mu_2 & 0 & (m^o_\nu)_{21} & (m^o_\nu)_{22} & (m^o_\nu)_{23} \\ 0 & 0 & -\mu_3 & 0 & (m^o_\nu)_{31} & (m^o_\nu)_{32} & (m^o_\nu)_{33} \end{array} \right) , \tag{3}$$

whose basis is $(-i\tilde{B}, -i\tilde{W}, \tilde{h}^0_u, \tilde{h}^0_d, \nu_{L_1}, \nu_{L_2}, \nu_{L_3})$, with $\tilde{h}^0_d$ being the neutral fermion from $\hat{L}_0$. The latter is guaranteed to be predominately a neutralino rather than neutrino, as the mass matrix clearly illustrates. As pointed out above, for small μ_i's the charged fermion states in the $\hat{L}_i$'s are essentially the physical states of e, μ and τ. Hence, $(\nu_{L_1}, \nu_{L_2}, \nu_{L_3})$ are essentially ν_e, ν_μ, ν_τ. The whole lower-right 3×3 block (m^o_ν) is, of course, zero at tree level. They are induced via 1-loop contributions to be discussed below.

We can write the mass matrix in the form of block submatrices:

$$\mathcal{M}_{\mathcal{N}} = \begin{pmatrix} \mathcal{M}_n & \xi^T \\ \xi & m^o_\nu \end{pmatrix} , \tag{4}$$

where $\mathcal{M}_n$ is the upper-left 4×4 neutralino mass matrix, ξ is the 3×4 block, and m^o_ν is the lower-right 3×3 neutrino block in the 7×7 matrix. The resulting (effective) neutrino mass matrix after block diagonalization is given by

$$(m_\nu) = -\xi \mathcal{M}_n^{-1} \xi^T + (m^o_\nu) \, . \tag{5}$$

The first term here corresponds to tree level contributions, which are, however, see-saw suppressed. The second term is the direct contribution, which, however, comes in only at 1-loop level. We will focus on all these contributions. When considering the small μ_i where the tree-level contribution is expect to be not strong than the 1-loop effect, it is clear that other 1-loop contributions to the ξ and $\mathcal{M}_n$ blocks would have only a secondary effect on (m_ν). The latter is hence not included in the present analysis.

LR-mixings for Squarks and Sleptons : The soft SUSY breaking part of the Lagrangian can be written as

$$\begin{aligned} V_{\rm soft} = {} & \epsilon_{ab} B_\alpha \, H_u^a \tilde{L}_\alpha^b + \epsilon_{ab} \left[A^U_{ij} \, \tilde{Q}^a_i H^b_u \tilde{U}^C_j + A^D_{ij} H^a_d \tilde{Q}^b_i \tilde{D}^C_j + A^E_{ij} H^a_d \tilde{L}^b_i \tilde{E}^C_j \right] + \text{h.c.} \\ & + \epsilon_{ab} \left[A^{\lambda'}_{ijk} \tilde{L}^a_i \tilde{Q}^b_j \tilde{D}^C_k + \frac{1}{2} A^{\lambda}_{ijk} \tilde{L}^a_i \tilde{L}^b_j \tilde{E}^C_k \right] + + \frac{1}{2} A^{\lambda''}_{ijk} \tilde{U}^C_i \tilde{D}^C_j \tilde{D}^C_k + \text{h.c.} \\ & + \tilde{Q}^\dagger \tilde{m}^2_Q \, \tilde{Q} + \tilde{U}^\dagger \tilde{m}^2_U \, \tilde{U} + \tilde{D}^\dagger \tilde{m}^2_D \, \tilde{D} + \tilde{L}^\dagger \tilde{m}^2_L \tilde{L} + \tilde{E}^\dagger \tilde{m}^2_E \, \tilde{E} + \tilde{m}^2_{H_u} \, |H_u|^2 \\ & + \frac{M_1}{2} \tilde{B}\tilde{B} + \frac{M_2}{2} \tilde{W}\tilde{W} + \frac{M_3}{2} \tilde{g}\tilde{g} \, , \end{aligned} \tag{6}$$

where we have separated the R-parity conserving ones from the RPV ones ($H_d \equiv L_0$) for the A-terms. Note that $\tilde{L}^\dagger \tilde{m}_{\tilde{L}}^2 \tilde{L}$, unlike the other soft mass terms, is given by a 4×4 matrix. Explicitly, $\tilde{m}_{L_{00}}^2$ is $\tilde{m}_{H_d}^2$ of the MSSM case while $\tilde{m}_{L_{0k}}^2$'s give RPV mass mixings.

The SVP also simplifies much the otherwise complicated expression for the mass-squared matrix of the scalar sectors. Firstly, we will look at the squarks sectors. The masses of up-squarks obviously have no RPV contribution. The down-squark sector, however, has interesting result. We have the mass-squared matrix as follows :

$$
{\cal M}_{D}^2 = \left(\begin{array}{cc} {\cal M}_{LL}^2 & {\cal M}_{RL}^{2\dagger} \\ {\cal M}_{RL}^2 & {\cal M}_{RR}^2 \end{array} \right) \;, \tag{7}
$$

where

$$
\begin{aligned}
{\cal M}_{LL}^2 &= \tilde{m}_{Q}^2 + m_D^\dagger m_D + M_Z^2 \, \cos 2\beta \left[-\frac{1}{2} + \frac{1}{3} \sin^2\theta_W \right] \;, \\
{\cal M}_{RR}^2 &= \tilde{m}_{D}^2 + m_D m_D^\dagger + M_Z^2 \, \cos 2\beta \left[-\frac{1}{3} \sin^2\theta_W \right] \;,
\end{aligned} \tag{8}
$$

and

$$
({\cal M}_{RL}^2)^T = A^D \frac{v_0}{\sqrt{2}} - m_D \, \mu_0^* \, \tan\beta - (\, \mu_i^* \lambda'_{ijk} \,) \, \frac{v_u}{\sqrt{2}} \;. \tag{9}
$$

Here, m_D is the down-quark mass matrix, which is diagonal under the parametrization adopted; $(\, \mu_i^* \lambda'_{ijk} \,)$ denotes the 3×3 matrix $(\;)_{jk}$ with elements listed [3] ; and $\tan\beta = \frac{v_u}{v_0}$. Apart from the first A^D term, the remaining terms in $({\cal M}_{RL}^2)^T$ are F-term contributions; in particular, the last term gives "SUSY conserving" [4] but R-parity violating contributions. Note that the full F-term part can actually be written together as $(\, \mu_\alpha^* \lambda'_{\alpha jk}) \, \frac{v_u}{\sqrt{2}}$ where the $\alpha = 0$ term gives the second term in RHS of Eq.(9), which is the usual μ-term contribution in the MSSM case. The latter is, however, diagonal, *i.e.* vanishes for $j \neq k$.

Next we go on to the slepton sector. From Eq.(6) above, we can see that the "charged Higgs" should be considered on the same footing together with the sleptons. We have hence an 8×8 mass-squared matrix of the following $4+3+1$ form :

$$
{\cal M}_{E}^2 = \left(\begin{array}{ccc} \widetilde{\cal M}_{LL}^2 & \widetilde{\cal M}_{RL}^{2\dagger} & \widetilde{\cal M}_{LH}^2 \\ \widetilde{\cal M}_{RL}^2 & \widetilde{\cal M}_{RR}^2 & \widetilde{\cal M}_{RH}^2 \\ \widetilde{\cal M}_{LH}^{2\dagger} & \widetilde{\cal M}_{RH}^{2\dagger} & \widetilde{\cal M}_{Hu}^2 \end{array} \right) \;; \tag{10}
$$

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[3] Note that we use this kind of bracket notations for matrices extensively in this letter. In this case, the repeated index i is to be summed over as usual, and hence dummy.

[4] However, it should be noted that existence of nonzero F-terms or electroweak symmetry breaking VEV's can be interpreted as a consequence of SUSY breaking.

where

$$
\begin{aligned}
\widetilde{\mathcal{M}}_{LL}^2 &= \tilde{m}_L^2 + m_L^\dagger m_L + (\mu_\alpha^* \mu_\beta) + M_Z^2 \cos 2\beta \left[-\frac{1}{2} + \sin^2\theta_W \right] \ , \\
\widetilde{\mathcal{M}}_{RR}^2 &= \tilde{m}_E^2 + m_E m_E^\dagger + M_Z^2 \cos 2\beta \left[- \sin^2\theta_W \right] \ , \\
\widetilde{\mathcal{M}}_{Hu}^2 &= \tilde{m}_{Hu}^2 + \mu_\alpha^* \mu_\alpha + M_Z^2 \cos 2\beta \left[\frac{1}{2} - \sin^2\theta_W \right] \ ;
\end{aligned} \tag{11}
$$

and

$$
(\widetilde{\mathcal{M}}_{RL}^2)^T = \begin{pmatrix} 0 \\ A^E \end{pmatrix} \frac{v_0}{\sqrt{2}} - \begin{pmatrix} 0 \\ m_E \end{pmatrix} \mu_0^* \tan\beta - (\, \mu_i^* \lambda_{i\beta k} \,) \frac{v_u}{\sqrt{2}} \ , \tag{12}
$$

$$
\widetilde{\mathcal{M}}_{RH}^2 = - (\, \mu_i^* \lambda_{i0k} \,) \frac{v_0}{\sqrt{2}} \ , \tag{13}
$$

$$
\widetilde{\mathcal{M}}_{LH}^2 = B_\alpha^* \ . \tag{14}
$$

Here, $m_L = \mathrm{diag}\{0, m_E\} \equiv \mathrm{diag}\{0, m_1, m_2, m_3\}$, where the three m_i's are masses from leptonic Yukawa terms as discussed above in relation to Eq.(2); and, again, $(\, \mu_i^* \lambda_{i\beta k} \,)$ denote a matrix (4×3) with elements given by $(\ )_{\beta k}$. Recall that for the small μ_i domain we focused on here in this letter, we have $m_E \simeq \mathrm{diag}\{m_e, m_\mu, m_\tau\}$. In fact, the k-th element in the 3-column-vector $\widetilde{\mathcal{M}}_{RH}^2$ in Eq.(13) can be written as simply as $\mu_k^* m_k$ (no sum). Similarly, the k-th element in the first row of the 4×3 matrix $(\widetilde{\mathcal{M}}_{RL}^2)^T$ in Eq.(12) can be written as $\mu_k^* m_k \tan\beta$ (no sum). The former is a $\ell_R^c h_u^-$ type, while the latter a $\ell_R^c h_d^-$ type ($h_d^- \equiv \ell_{L_0}$), mass-squared term. Or, to better illustrate the common flavor structure, one can put the full F-term part of Eq.(12) as $- (\, \mu_\alpha^* \lambda_{\alpha\beta k} \,) \frac{v_u}{\sqrt{2}}$.

For the sake of completeness, we give here also the correponding sneutrino-Higgs mass squared matrix as

$$
\mathcal{M}_S^2 = \begin{pmatrix} \tilde{m}_L^2 + (\mu_\alpha^* \mu_\beta) + M_Z^2 \cos 2\beta \left[\frac{1}{2} \right] & -(B_\alpha^*) \\ -(B_\alpha) & \tilde{m}_{Hu}^2 + \mu_\alpha^* \mu_\alpha + M_Z^2 \cos 2\beta \left[-\frac{1}{2} \right] \end{pmatrix} \ . \tag{15}
$$

The B_α entries may also be considered as a kind of LR mixings. The RPV B_i's do in fact contribute to neutrino mass, as discussed below.

We would like to emphasize that the above scalar mass results are complete — all RPV contributions, SUSY breaking or otherwise, are included. The simplicity of the result is a consequence of the SVP. Explicitly, there are no RPV A-term contributions due to the vanishing of VEV's $v_i \equiv \sqrt{2}\langle \hat{L}_i \rangle$. The Higgs-slepton results given as in Eqs.(10) and (15) are admittedly not very useful for doing scalar physics. They contain redundancy of parameters and hide the unphysical Goldstone state. The large number of parameters involved, anyway, makes it difficult to learn much about the scalars. However, for the purpose of analyzing the neutrino mass contributions, they are good enough. Hence, we will refrain from further laboring on the algebra here and leave it for later studies.

Neutrino Mass Contributions : Let us get back to RPV contribution to neutrino mass. From Eq.(3), one neutrino state get a tree-level mass. The see-saw suppressed contribution [see Eqs.(4) and (5)] is given by [3]

$$(m_\nu)^{\rm tree}_{ij} \sim \frac{-v^2 \cos^2\!\beta \; ({g_2}^2 M_1 + {g_1}^2 M_2)}{2\mu_0 \left[2\mu_0 M_1 M_2 - v^2 \sin\!\beta \cos\!\beta \; (g_2^2 M_1 + g_1^2 M_2)\right]} \;\; \mu_i \mu_j \; . \tag{16}$$

This is illustrated diagrammatically in Fig. 1. At 1-loop level, there are many contributions.

A typical 1-loop neutrino mass diagram has two couplings of scalar-fermion-neutrino type. With the two couplings being λ'-type, we have a quark-squark loop as shown in Fig. 2. Here, a LR squark mixing is needed. From Eqs.(7) and (9), we have the result, here written in three parts : firstly the familiar one

$$(m_\nu)^{\rm sqA}_{ij} \sim \frac{3}{16\pi^2} \, \frac{m_{D_h} m_{D_k}}{M_{\tilde{d}}^2} \, \lambda'_{ihk} \lambda'_{jkh} \; [A_d - \mu_0^* \tan\!\beta] \; , \tag{17}$$

where $M_{\tilde{d}}$ denote an average down-squark mass, and A_d being a constant (mass) parameter representing the "proportional" part of the A-term, namely $A^D \frac{v_0}{\sqrt{2}} = A_d \, m_D + \delta A^D \; \frac{v_0}{\sqrt{2}}$, and m_{D_h} is the h-th diagonal element of the matrix m_D (*i.e.* the quark mass); next, the "proportionality" violating part

$$(m_\nu)^{\rm sq\delta A}_{ij} \sim \frac{3}{16\pi^2} \, \frac{m_{D_h}}{M_{\tilde{d}}^2} \, \lambda'_{ihl} \lambda'_{jkh} \left[\delta A^D_{kl} \frac{v_0}{\sqrt{2}}\right] \qquad (\; i \longleftrightarrow j \;) \; , \tag{18}$$

which is typically expected to be suppressed in many SUSY breaking scenarios and neglected; and, finally, the part due to the new RPV LR mixings,

$$(m_\nu)^{\rm sq\not{R}}_{ij} \sim -\frac{3}{16\pi^2} \, \frac{m_{D_h}}{M_{\tilde{d}}^2} \, \lambda'_{ihl} \lambda'_{jkh} \left[\mu_g^* \lambda'_{gkl} \, \frac{v_u}{\sqrt{2}}\right] \qquad (\; i \longleftrightarrow j \;) \; . \tag{19}$$

The $(\; i \longleftrightarrow j \;)$ expression denote symmetrization with respect to i and j. It is interesting to note that the last result contains no SUSY breaking parameter in the LR mixings.

Similar to the quark-squark loop, a lepton-slepton loop with two λ-type coupling, as shown in Fig. 3, generates neutrino mass, in the presence of LR slepton mixings. Using Eqs.(10) and (12), again we split the result into the different parts : the familiar one from the "proportional" part of the A-term,

$$(m_\nu)^{\rm slA}_{ij} \sim \frac{1}{16\pi^2} \, \frac{m_h m_k}{M_{\tilde{\ell}}^2} \, \lambda_{ihk} \lambda_{jkh} \; [A_e - \mu_0^* \tan\!\beta] \; , \tag{20}$$

where $M_{\tilde{\ell}}$ denote an average charged slepton mass, and A_e the constant (mass) parameter with $A^E \frac{v_0}{\sqrt{2}} = A_e \, m_E + \delta A^E \; \frac{v_0}{\sqrt{2}}$, (recall that m_h's are diagonal element of m_E and essentially the mass of the charged lepton); the "proportionality" violating part

$$(m_\nu)^{\rm sl\delta A}_{ij} \sim \frac{1}{16\pi^2} \, \frac{m_h}{M_{\tilde{\ell}}^2} \, \lambda_{ihl} \lambda_{jkh} \left[\delta A^E_{kl} \frac{v_0}{\sqrt{2}}\right] \qquad (\; i \longleftrightarrow j \;) \; ; \tag{21}$$

and the part due to the new RPV LR mixings,

$$(m_\nu)_{ij}^{\mathrm{sl}\not{R}} \sim -\frac{1}{16\pi^2}\,\frac{m_h}{M_{\tilde{\ell}}^2}\,\lambda_{ihl}\lambda_{jkh}\,\left[\mu_g^*\lambda_{gkl}\,\frac{v_u}{\sqrt{2}}\right] \qquad (\, i \longleftrightarrow j \,)\,. \tag{22}$$

However, the above is not yet the full result for the type of contributions. We have emphasized throughout the letter the systematic treatment of making no *a priori* distinction between the $\hat{L}_i$'s and $\hat{H}_d$. The latter is denoted as $\hat{L}_0$ and treated as a 4-th leptonic flavor. In Eq.(12), the last term admitted a $\beta = 0$ part the neutrino mass contribution of which has not been included in the above analysis of the lepton-slepton loop parallel to the quark-squark loop. The corresponding result is simply given by setting k to 0 in Eq.(22), which may then be simplified to

$$(m_\nu)_{ij}^{\mathrm{slZ}} \sim -\frac{1}{16\pi^2}\,\frac{\sqrt{2}}{v_0}\,\frac{m_j^2}{M_{\tilde{\ell}}^2}\,\lambda_{ijl}\,\left[\,\mu_l^* m_l\,\tan\!\beta\,\right] \qquad (\, i \longleftrightarrow j \,)\,. \tag{23}$$

The contribution corresponds to the SUSY analog of the Zee neutrino diagram [12], as discussed in Ref. [5]. We illustrate the contribution and its Zee analog in Fig. 4. A careful examination of Fig. 3 shows that one cannot get any more new neutrino mass diagram by replacing some other λ_{ijk} flavor indices with a 0. Hence, we have completed the listing of the two-λ-loop contributions.

The only other couplings involving a neutrino are the gauge couplings and the bilinear μ_i's. The effect of the latter has been considered in the tree-level see-saw. Putting two gauge couplings together, we do have a 1-loop neutrino mass diagram, with scalars and gauginos running in the loop. The charged loop does not work, while a neutral loop could do (see Fig. 5) when there is a Majorana-like sneutrino mass term. The latter contribution is first pointed out in Ref. [13]. In fact, Majorana-like sneutrino mass is where the required two units of lepton number violation come in. The former may be interpreted as a result of splitting in mass of the sneutrino and anti-sneutrino due to R-parity violation. Following our general approach in this letter, we illustrate this in Fig. 6. It is clear from the figure that it involves the SUSY breaking and RPV parameters B_i's, as shown is Eq.(15) above, and is see-saw suppressed (*cf.* Fig 1), unless the B_i's happen to be at the SUSY scale despite small μ_i's. Consistent with the level of treatment in this letter, we refrain from going into further discussions of this contribution.

There is also possible to have a 1-loop diagram with one λ-coupling and one gauge coupling. This is illustrated in Fig. 7. It is not *a priori* clear that this contribution is negligible on our level of treatment. However, one has to note that after symmetrizing with respect to i and j, the sign flip in λ_{jik} coupling of the diagram gives a prefect cancellation for degenerate sleptons $\tilde{\ell}_{L_i}$ and $\tilde{\ell}_{L_j}$. Hence, the contribution from Fig. 7 is suppressed by the small degeneracy violation and really negligible.

Concluding Remarks : From the above systematic analysis, it is obvious that we have discussed and given explicit formulae for all neutrino mass contributions up to the level of direct 1-loop contribution, for the complete theory of SUSY without R-parity. We have also given a description of the full squark and slepton masses. The latter is useful for analyzing other aspects of phenomenology, particularly those related to LR mixings such as fermion electric dipole moment and flavor changing neutral current processes. The successful simple description here illustrates well the effectiveness of the formulation (SVP) adopted.

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REFERENCES

[1] For an updated review, see V. Barger, hep-ph/0003212.

[2] M. Drees, S. Pakvasa, X. Tata, and T. ter Veldhuis, Phys. Rev. **D57** (1998) 5335; S. Rakshit, G. Bhattacharyya, and M. Raychaudhuri, Phys. Rev. **D59** (1999) 091701 ; E. Chun, S. Kang, C. Kim, and U. Lee, Nucl. Phys. **B544** (1999) 89; J. Romao, M. Diaz, M. Hirsch, W. Porod, and J. Valle, Phys. Rev. **D61** (2000) 071703.

[3] O.C.W. Kong, Mod. Phys. Lett. **A14** (1999) 903 .

[4] A. Abada and M. Losada, hep-ph/9908352.

[5] K. Cheung and O.C.W. Kong, hep-ph/9912238, Phys. Rev. **D61** (2000) *to be published.*

[6] Super-Kamiokande Collaboration, Y.Fukuda *et al.*, Phys. Rev. Lett. **81** (1998) 1562; P. Lipari, hep-ph/9904443; G.L. Fogli, E. Lisi, A. Marrone, and G. Scioscia, Phys. Rev. **D59** (1999) 033001.

[7] E.J. Chun and S.K. Kang, Phys. Rev. **D61** (2000) 075012.

[8] M. Bisset, O.C.W. Kong, C. Macesanu, and L.H. Orr, Phys. Lett. **B430** (1998) 274 ; hep-ph/9811498, Phys. Rev. **D** (2000) *to be published.*

[9] Y.-Y. Keum and O.C.W. Kong, *manuscript in preparation*

[10] Y.-Y. Keum and O.C.W. Kong, *work in progress.*

[11] L.E. Ibáñez and G.G. Ross, Nucl. Phys. **B368** (1992) 3.

[12] A. Zee, Phys. Lett. **93B** (1980) 389 .

[13] Y. Grossman and H.E. Haber, hep-ph/9906310.

Figure captions :

Fig. 1 — Neutrino mass from tree-level see-saw.

Fig. 2 — Neutrino mass from quark-squark loop.

Fig. 3 — Neutrino mass from lepton-slepton loop.

Fig. 4 — SUSY Zee diagram for neutrino mass. The Zee analog interpretation noted in the brackets.

Fig. 5 — Gaugino-sneutrino loop requiring a Majorana-like sneutrino mass insertion.

Fig. 6 — See-saw diagram for Majorana-like sneutrino mass.

Fig. 7 — Diagram with a λ- and a gauge coupling.

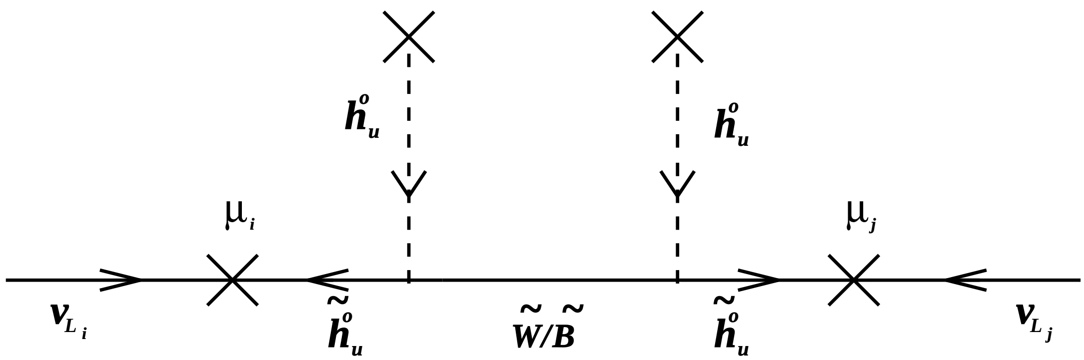

FIG. 1. Neutrino mass from tree-level see-saw.

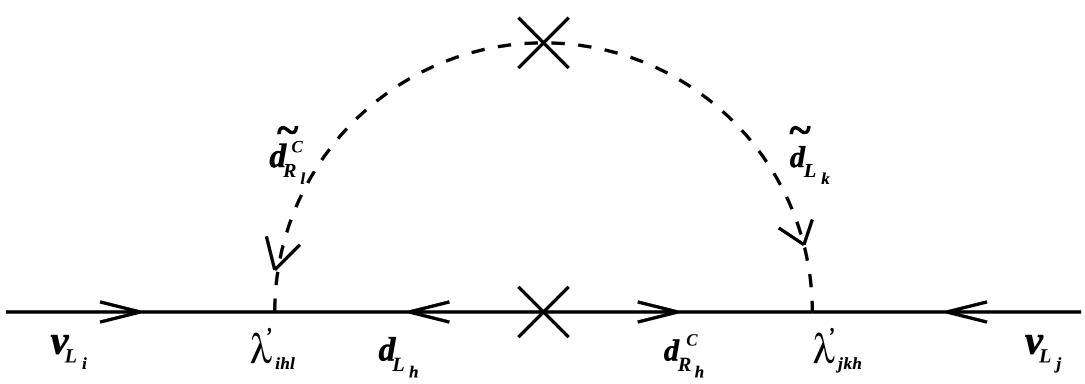

FIG. 2. Neutrino mass from quark-squark loop.

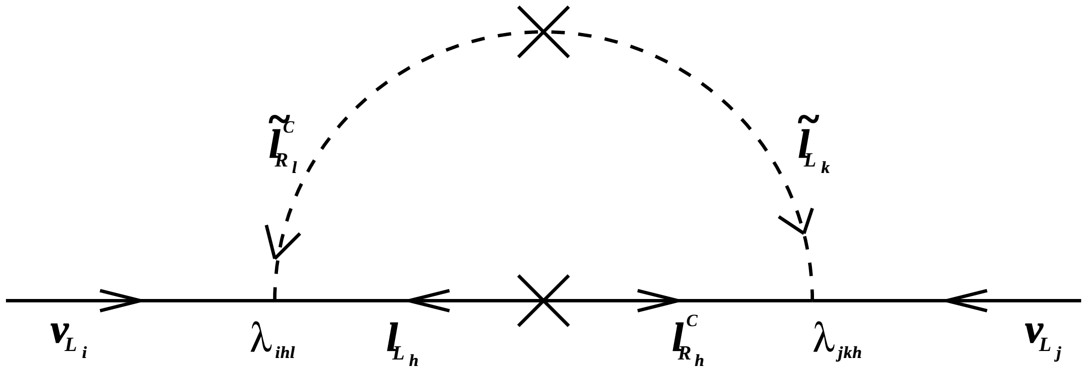

FIG. 3. Neutrino mass from lepton-slepton loop.

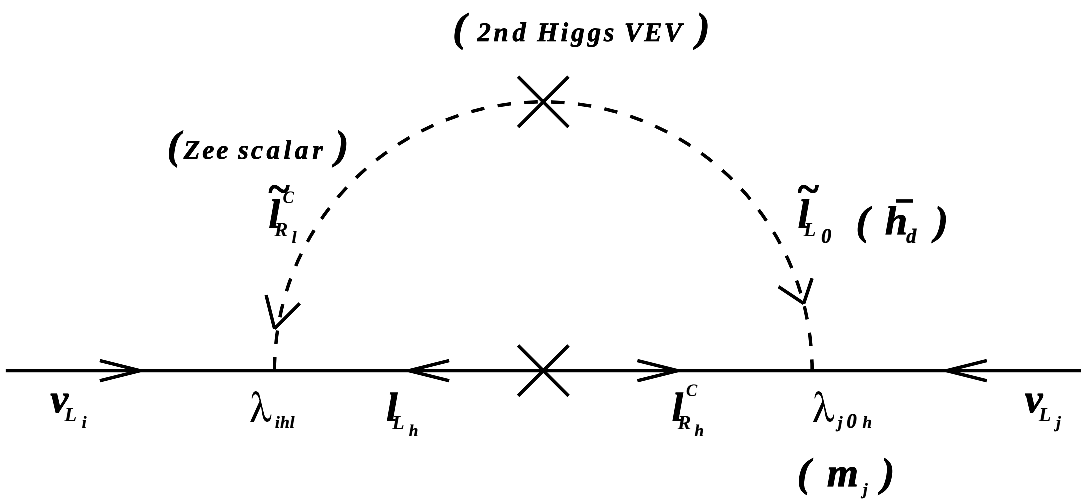

FIG. 4. SUSY Zee diagram for neutrino mass. The Zee analog interpretation noted in the brackets.

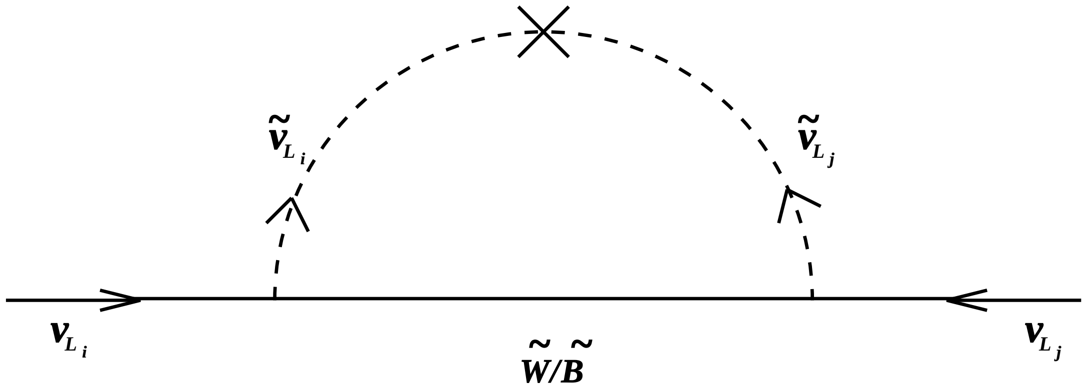

FIG. 5. Gaugino-sneutrino loop requiring a Majorana-like sneutrino mass insertion.

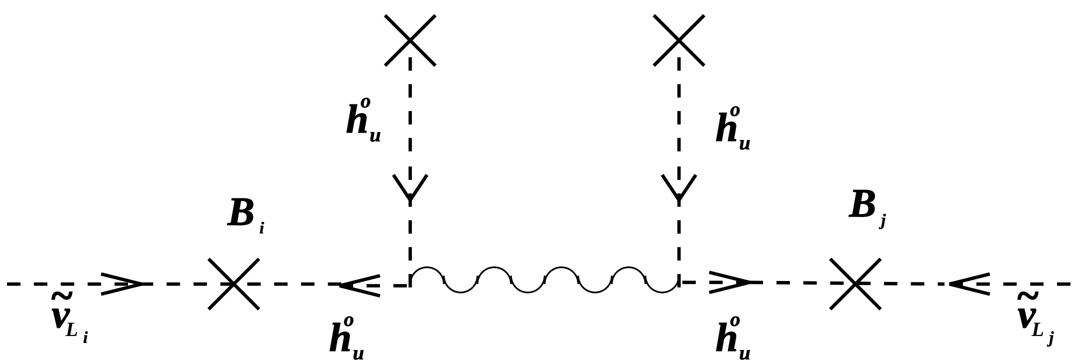

FIG. 6. See-saw diagram for Majorana-like sneutrino mass.

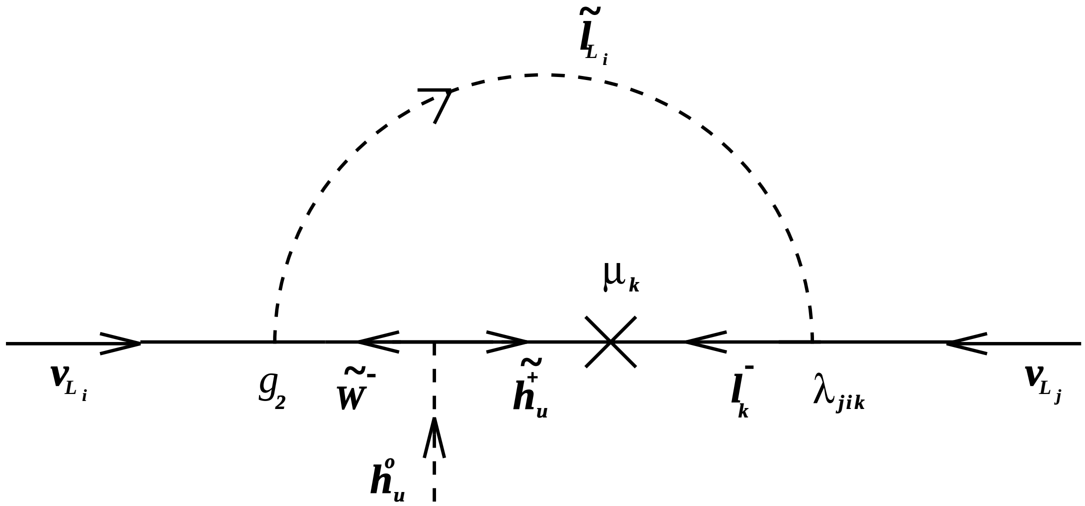

FIG. 7. Diagram with a λ- and a gauge coupling.